

Comparing Slopes

Explore the Mathematics

Definitions of Slope

Slope is a way to describe the steepness, or tilt, of a line. Intuitively a steeper hill will have a greater slope than a hill that rises more gradually.

Slope is defined as the ratio of the change in y to the corresponding change in x . We can use the “slope formula,” $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are any two points on the line to calculate slope. For a line given in the form of $y = mx + b$ (slope intercept form), m is the slope of that line.

Slopes of Horizontal and Vertical Lines

Horizontal lines have a slope of zero. Every point on a horizontal line has the same y -coordinate. In other words, there is no change in y . When calculating the slope of a horizontal line, one divides zero by some change in x , obtaining a result of zero.

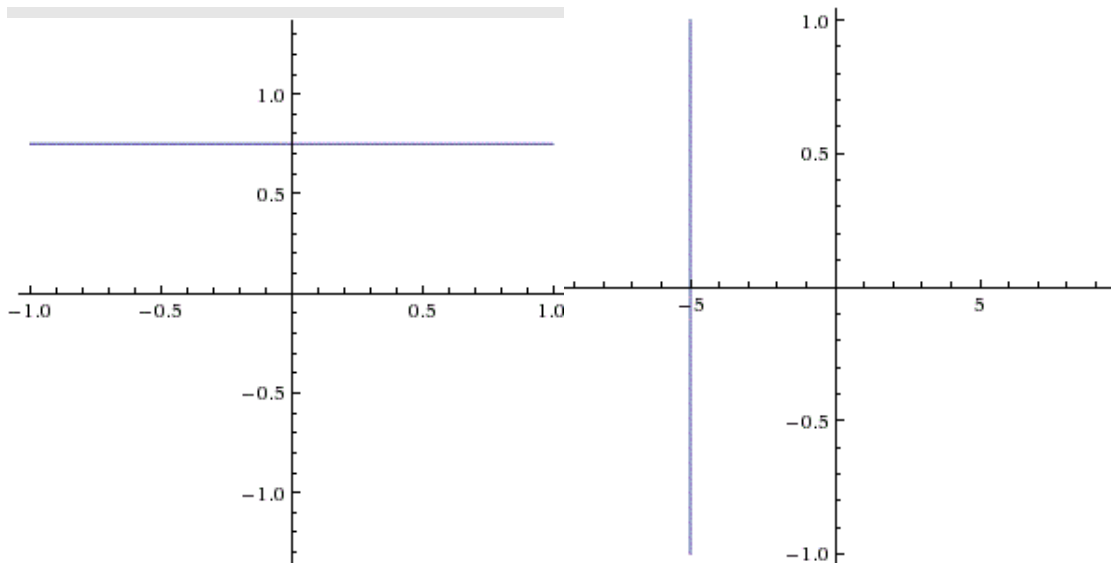
For example:

$$\text{Line: } y = \frac{3}{4} \quad \text{Points: } (-5, \frac{3}{4}) \text{ \& } (-2, \frac{3}{4}) \quad \text{Slope: } \frac{\frac{3}{4} - \frac{3}{4}}{-2 - -5} = \frac{0}{3} = 0$$

Vertical lines have an undefined slope. Every point on a vertical line has the same x -coordinate, meaning that there is no change in x . When calculating the slope of a vertical line, one divides the change in y by zero; mathematically, division by zero is not defined, making the slope of any vertical line undefined. For example:

$$\text{Line: } x = -5 \quad \text{Points: } (-5, 2) \text{ \& } (-5, 6) \quad \text{Slope: } \frac{6 - 2}{-5 - -5} = \frac{4}{0} = \text{undefined}$$

If slope is thought of as the steepness of a hill, a slope of 3 would be a steep hill, a slope of 1 would be less steep, $\frac{1}{2}$ even less steep, all the way down to a slope of 0, which would be a hill with no steepness, that is a horizontal line. Going the other way, a hill with a slope of 10 would be much steeper than a hill with a slope of 3, a slope of 1000 would be steeper still, all the way to a perfectly vertical hill with infinite steepness. Instead of saying that the vertical hill has an infinite steepness, mathematically we say that its steepness (or slope) is undefined.

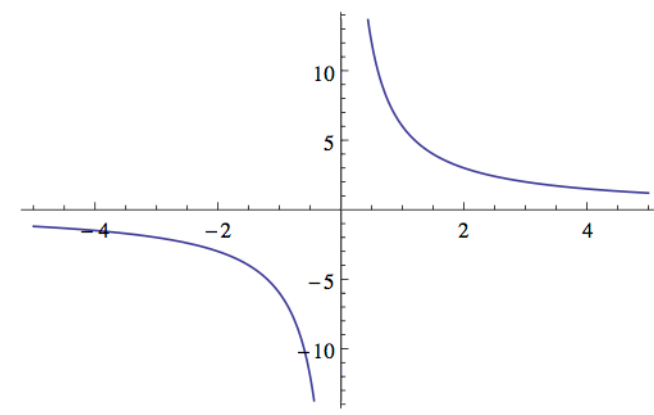


Dividing by Zero

There are several ways to explain why dividing by zero is undefined. If $6/0$ is defined, then it must equal some number. Call that number x . Then $6/0 = x$. Multiply both sides of the equation by zero, making $6 = 0x$. Since any number multiplied by zero is zero, there is no number x that will make this equation true.

From the perspective of division as grouping, $6/2$ can be thought of as 6 objects divided into 2 groups. The answer is 3 because each of your two groups would contain 3 objects. $6/0$ would mean dividing 6 objects into zero groups, which isn't possible – you need at least one group to contain the objects.

Finally, we could attempt to formally define $6/0$ as the limit of $f(n) = 6/n$ as n approaches zero. (This method of thinking is common in calculus.) The limit is undefined because there is not a single value for the function as you approach from the left or from the right. The graph below helps make it clear that the function is not defined at $x = 0$.



Connecting to Algebra

Slopes are a fundamental idea of algebra courses as well as courses that build off of algebra. In this video students discuss slopes using various representations, including graphs and equations, which is a focus of the algebra curriculum. They also discuss slope as both steepness and rates of change, two key articulations of the concept of slope addressed by most algebra texts.

Connecting to the Common Core Standards

8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).