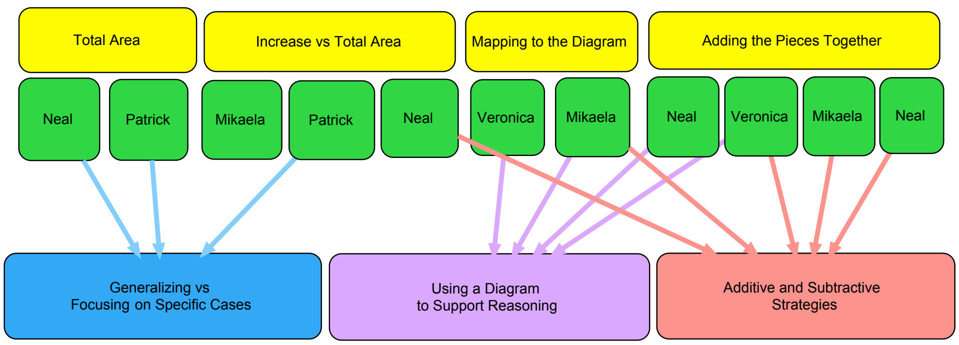
Increasing Area
Explore Themes in Student Thinking

Throughout the video, students describe their reasoning about how to construct a general expression for an increase in area of a square plot of land. Stepping back from the chronology of these ideas, we can see three main themes in the students' thinking. Students reason either by generalizing about the side lengths or relying on specific cases, by relying on the diagram, and by using either additive or subtractive strategies.

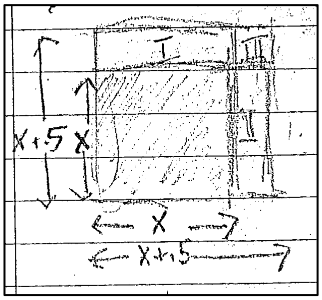


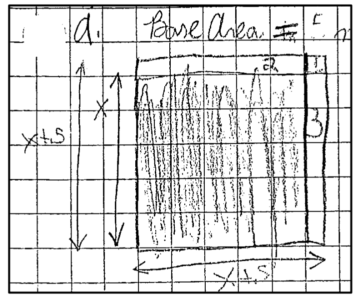
Theme 1: Generalizing vs. Focusing on a Specific Case

The goal of this activity is to find a general solution for how much the area of a square plot increases when the side length increases by 0.5 miles. To do so, several of the students in the group use "x" to represent the original side length of the square. Neal is the first to use x in this way. He says, "[the solution is] parentheses x plus 0.5 squared." Mikaela and Veronica tacitly agree with his use of x, and both discuss various solutions involving x; Mikaela talks about "x plus 0.5" and Veronica talks about "x times 0.5." In each of these expressions, the students unproblematically use x to represent a side length. This type of reasoning is precisely what is called for in the activity.

In contrast, throughout the discussion, one student, Patrick, focuses on the increase in area in terms of specific cases. In particular, he checks the other students' proposed solutions by first calculating the area increase for a side length of 2 miles. He says, "Let's use an example first… let's use 4 [2 squared] because that's the easiest and I know the answers." Further, he disagrees with Neal's solution because "that wouldn't really make sense if it [the side length] was 2" and agrees with Mikaela's answer because "our equation works because that's the exact number that we had [when we calculated for the side length 2]." From one perspective Patrick's focus on using a side length of 2 rather than "x" is problematic; the purpose of the problem is to develop fluency with using x to represent any potential side length and his method is not generalizable to other cases. However, from another perspective Patrick's approach allows him to examine the plausibility of his peers' solutions; the algebraic expression must yield a reasonable answer for the specific cases they have already calculated. Although in this case Patrick does not seem to use his case-based reasoning to *generate* generalized solutions, he does use it productively to *check* generalized solutions.

Theme 2: Using a Diagram to Support Reasoning
Prior to their discussion of a generalizable formula, the students have created a diagram showing how the area of the square plot increases when the side length is increased. Below are two examples from the students' work.





The diagrams illustrate three new areas of the plot: two long, rectangular pieces on the sides of the square and one "corner" piece. Several students subsequently use the diagram to reason about the increased area. For example, Veronica describes the three pieces they need to account for in their expression saying, "so it's one, three, and two [on the diagram]." Similarly, Neal acknowledges that the 0.5 squared term alone cannot be the solution because "you'd only find the area of one [of the pieces] I think."

Mikaela goes even farther than Veronica and Neal; she describes a one-to-one correspondence between the three new areas in the diagram and the terms in her expression for the increased area. For example, she says, "because look, this is x times 0.5 and this is x times 0.5… That's it for both of it [long sides]. And then 0.5 times 0.5 is the little corner." She explicitly justifies particular parts of her expression by describing how they account for the three new pieces of area. It is not clear whether these students are using the diagram to generate their expressions or if they are generating their expressions independently and then checking them against the diagram. The distinction is not important here; either way the diagram serves as an important resource for the students in reasoning about the expression for the increased area of the square.

Theme 3: Additive and Subtractive Strategies
The students in the video are attempting to "find an expression using x that gives the amount by which the area of Westville *increased.*" Mathematically, there are two ways to calculate the increased area: 1) add together the three parts of the area that result from the additional side length or 2) subtract the original area from the total area of the square with the increased side length. Students in the video use both methods. Mikaela seems to use an additive strategy saying, "this [term] would find this half and this half [the side pieces]… and this [term] would find the corner." Mikaela constructs the expression for the increase in area from the three parts.

In contrast, Neal uses a strategy that begins with the total area and subtracts off the original area. He says "you just want the growth you just drop out that last x [squared] which is the original" Additionally, he comments on Patrick's solution by saying "technically you want to leave that 4 squared [the original area] out." This strategy may stem from Neal's original misinterpretation of the problem as being about the total area. It is after Mikaela emphasizes the word "increased" in the problem several times that Neal suggests subtracting the original area from the total area.

Later parts of the problem will ask students to compare these two methods – the additive and the subtractive. However, at this point in the video it is not clear that students explicitly recognize that they are taking two different approaches.