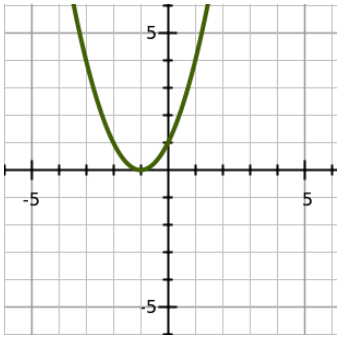
Graphing Quadratics
Explore the Mathematics

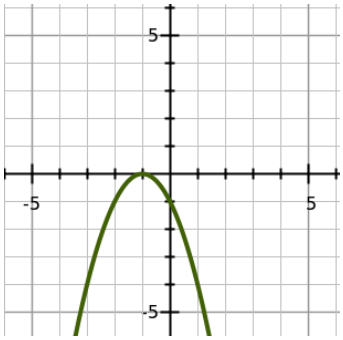
Key Properties of Quadratic Functions

Functions of the form $f(x) = ax^2 + bx + c, a \neq 0$ are quadratic. There are several ways quadratic functions can be expressed. Different forms of quadratic functions highlight different aspects of the function. For example, when expressed in vertex form $f(x) = a(x-h)^2 + k$, where (h, k) is the vertex, the coordinates of the vertex are highlighted and when expressed in factored form $f(x) = a(x-x_1)(x-x_2)$, the roots (x_1 and x_2) are highlighted.

Graphs of quadratic functions, otherwise known as parabolas, have several distinct features. They have vertical symmetry as well as a vertex, which lies on the vertical line of symmetry. The vertex is a minimum point if the graph opens upward. The vertex is a maximum point if the graph opens downward. We can tell whether the graph opens upward or downward by looking at the sign of the coefficient of the squared term, a. As illustrated in the following graphs, if a is positive, the graph opens upward and if a is negative, the graph opens downward.

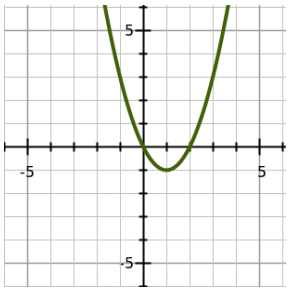


$a > 0$

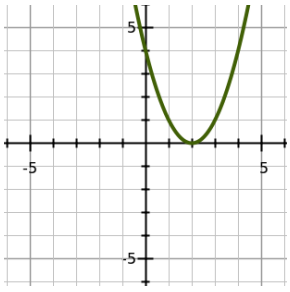


$a < 0$

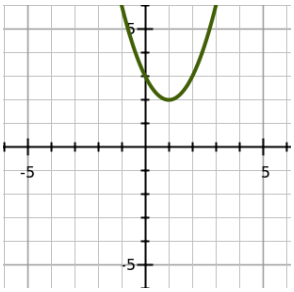
Quadratic functions have at most two real roots, the points where the graph crosses the x-axis. If the graph has two real roots, it crosses the x-axis at those values. If it has one real root, the vertex lies on the x-axis. If the function has no real roots, the graph will not cross the x-axis.



Two real roots



One real root



No real roots

Sherin, M. G., Russ, R. R., Walkoe, J., & Dyer, E. (2023). Algebra classroom video cases. Freezing Time Research Group. https://www.freezingtime.sesp.northwestern.edu/videocases. © 2023. Licensed under Creative Commons AttributionNonCommercialShareAlike 4.0 International.

Connecting to Algebra
This video shows students moving beyond linear functions and reasoning about quadratic functions. This activity challenges students to combine their thinking about graphs with their knowledge of properties of quadratic functions. One challenge for students in this problem is that the vertex is not among the points they are initially asked to plot. This necessitates that they reason about a parabola's symmetry and shape as well as reasoning about what other points would be helpful to plot in order to sketch a graph of the function.

Connecting to the Common Core Standards
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.