# Sequences

## Explore the Mathematics

The heights of the towers in the problem students are working on represent an arithmetic sequence. An arithmetic sequence is a list of numbers that have a common difference, in other words, in which the difference between any two terms is constant. For example,

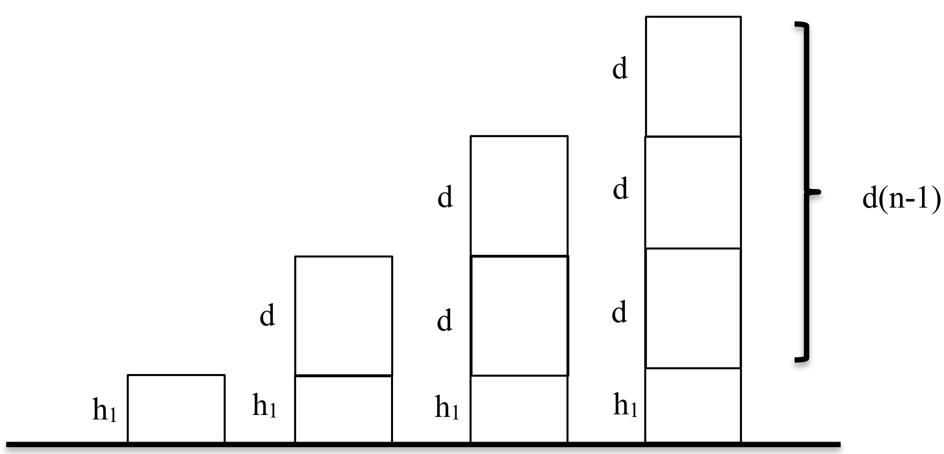
2, 5, 8, 11, …

is an arithmetic sequence with an initial term *($h_1$)* of 2, and a common difference *(d)* of 3. The students are asked to find a formula to express the height of the tallest tower, $h_n$, which is the $n^{th}$ term in the arithmetic sequence.

To do so, we can use a *recursive formula* or an *explicit formula*. A *recursive formula* allows us to find the value of any term in a sequence, given that we know the value of the previous term. This method is useful if we want to find the 6th term of the sequence above. We already know the 4th term, so we simply have to add three to find the 5th term (14), and then add three again to find the 6th term (17).

An *explicit formula* allows one to determine the value of any term in a sequence based on knowing just the value of the first term and the common difference. In general, the explicit formula for an arithmetic sequence can be written as:

$h_n = h_1 + d(n - 1)$, where
$d$ = the common difference
$h_1$ = the value of the first term (also called the initial term)
$n$ = the number of terms we are considering
$h_n$= the value of the $n^{th}$ term (the final term we are considering).




| n | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $h_n$ | $h_1$ | $h_1 + d$ | $h_1 + 2d$ | $h_1 + 3d$ |

The (n – 1) in the formula may be surprising at first. It makes sense to use multiplication to indicate the repeated addition of d, but why (n – 1) instead of just *n*? For an arithmetic sequence, you do not need to add the common difference to find the first term; you start adding it with the second term, meaning that you have added it once for the 2nd term, twice for the 3rd term, etc.

Note that, in the case that $h = d$ (that is, the first term and the difference are the same), the formula simplifies considerably:

$$h_n = h + d(n-1)$$
$$h_n = d + d(n-1)$$
$$h_n = d + dn - d$$
$$h_n = dn$$

In that case, the first term and the common difference are the same, so the value of any term is simply the value of the first term times the number of terms in your sequence.

*Connecting to Algebra*
In this video, students are reasoning about sequences. They are trying to find a general formula for the height of the tallest tower in an algebraic sequence. The students are exploring ideas using various algebraic ways of thinking. They model the situation using centimeter cubes and try to come up with a formula to generalize their model. The students move back and forth between thinking about the problem spatially, using the centimeter cubes, and symbolically. They find that their model both facilitates and constrains their thinking about the problem symbolically.

*Connecting to the Common Core Standards*
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*