Solving a Quadratic Equation
Explore Themes in Student Thinking

Throughout the video, students reason about equivalence in solving algebraic equations. In particular students discuss how to maintain equivalence and demonstrate different strategies for testing equivalence.

Theme 1: Reasoning about Maintaining Equivalence
Maintaining equality is a central part of solving and manipulating equations. In this video, students show evidence of reasoning about how to maintain equivalence. In particular, students regularly use the idea that performing the same operation on each side of the equation maintains equality; students talk about the need to operate on "both sides" of the equation. For example in Jeff's initial presentation of his group's solution he describes how "you add 4x to both sides… subtract 1 from both sides… then square root both sides." Damian explicitly makes the connection between the "both sides" language and the idea of equality. He says, "if you do something to one side, you do the exact same thing to the other side to make both sides equal." During the video students use this reasoning when talking about both taking the square root and following the reverse order of operations.

While reasoning about "both sides" is important for solving equations and is common language in the math classroom, this idea leads Damian in an unproductive direction. When Ian suggests that taking the square root would produce both a plus and a minus, Damian applies this reasoning to "both sides" of the equation. He says "wouldn't that work both ways?... if it's… plus or minus on that side it would have to be plus or minus on that side too." Damian then concludes that "if both sides plus or minus they kind of cancel out." Here, the idea of maintaining equivalence by performing the same operation on both sides (i.e. adding a plus and a minus) leads Damian to think about cancelling out, which results in him proposing an incorrect solution.

Theme 2: Strategies for Demonstrating Equivalence
In addition to maintaining equivalence in a solution path, students throughout the video also present strategies for demonstrating, or testing, the equivalence of various different solutions. The need to demonstrate equivalence between solutions gives rise to both concerns about the number of solutions and concerns about order of operations. For example, Quinn suggests that it does not matter whether you divide or square root first because "if you divided by 4 first [instead of square rooting]… then [the answer] is 1.5. It's the same thing." As Luke points out, Quinn's strategy involves using a "specific example." Becky uses a similar strategy for deciding the number of solutions. She says, "I substituted negative 1.5 and 1.5 back in the problem and they both worked out." Quinn and Becky rely on specific cases to show equivalence of methods and solutions.

In contrast, some students demonstrate equivalence more generally by describing the nature of the operations they are performing. For example, Joe uses the definition of squaring to explain why the order in which students square root and divide is irrelevant. He says, "if you square something, it's just multiplying by itself twice… so [squaring] is still just a form of multiplication. So it doesn't really matter." Nina similarly offers generalizable reason for the existence to be two equivalent solutions. She describes how "if it's a parabola…then it's symmetrical" and thus has two opposite solutions. Both Joe and Nina rely on general characteristics of quadratic equations to argue for the equivalence of two different solutions.

