# Graphing Quadratics
## Explore Themes in Student Thinking

Throughout the video, students express a range of mathematical ideas about graphing and parabolas. Students demonstrate a focus on the origin, discuss individual points versus the shape of the graph, and describe their reasoning about sameness.

*Theme 1: Focusing on the Origin*
The original problem provides several points from which to find an equation and graph, but the origin (0,0) is not among the points given. However, students seem to quickly decide that knowing whether the origin is included in the graph is important to understanding the parabola. Early in the video, Alessandra asks whether "it's supposed to go through zero?" Several students then turn to the task of figuring out what happens to the graph at zero. They quickly discover that (0,0) is in fact part of the rule ($n^2+n$): "exactly it'll go zero zero" and "x zero is y zero." Later when the teacher asks Chrissy to "enhance" her existing graph, Chrissy agrees with other students who suggest "we can start at… zero… zero squared plus zero… the point would be (0,0). So it'd be right at the origin." Knowing the behavior of the graph at zero is important for students working on this problem.

However, the fact that the graph goes through the origin seems to hinder some students in reasoning about the location of the vertex. For example, one student says "zero's where it bounces… it should pass through." Another student suggests, "wouldn't it [bounce] at zero, because zero squared plus zero is zero." These students tacitly assume that "passing through" the origin means the origin is the vertex. While incorrect in this case, this assumption is likely consistent with many of their experiences in math class; when first learning about parabolas the vertex is often (0,0). The salience of the origin as a point on the graph, combined with the fact that the vertex of this particular parabola is very close to the origin, may make it difficult for some students to reason independently about the vertex.

*Theme 2: Reasoning about Graphs as a Set of Points verses Graphs as a Whole*
When reasoning about this problem, some students focus on the location of individual points while others seem to consider the shape of a graph more holistically. In addition to focusing on the point (0,0) several students attempt to identify and plot other values that follow the rule. Nico focuses on two points, x=-3 and x=3, and Chrissy repeats a calculation of x=-3. Another student suggests that Chrissy plot x=1 saying "try 1 squared plus 1." In contrast, when reasoning about the graph as a whole rather than as a collection of points, several students use gestures that are helpful for communicating their thinking. One students says, "it would be more like this" and another student says "look its going like (swish)" while making the shape of a parabola in the air. Students' extended discussion about the symmetry of the graph also reflects a focus on the shape rather than on individual points.

Not only do different students reason differently about the graph, the same student may reason both about individual points and about the graph as a whole. It is precisely the mismatch between her point-based reasoning and her holistic, shape-based reasoning that gives rise to Chrissy's initial confusion and resulting discussion in the video. After she has plotted the points on the overhead, she says "I don't know, I think it was supposed to be a parabola, but I didn't exactly know how to make it a parabola." She is unable to reconcile what she knows to be the case from plotting the points and her belief that the shape of the graph should be a parabola.

*Theme 3: Reasoning about Sameness*

Intuitive notions of sameness are a useful starting point for thinking about the mathematical concept of symmetry. Throughout early parts of the video, students repeatedly use the language of sameness to make sense of the shape of parabolas. For example, Chrissy states that parabolas should have "the same thing" on one side as they have on the other and should "match up." Alessandra echoes this reasoning saying "if you cut it in half and you flip it over, it would look the same." In addition, she finds it problematic that in the given set of points, "[the y-coordinate of] -3 and 3 aren't the same." Here, Chrissy and Alessandra are thinking about sameness in relation to the vertical distance from the origin. Their focus on the lack of sameness of the y-coordinates makes it difficult for them to understand the shape of the parabola.

Other students in the class are able to use notions of sameness (or lack of sameness) to help them make sense of the parabola. For example, Nico says, "The y-coordinate numbers should be the same for reverse, like -3 and 3 should both be 12." Rather than this being a problem as it was for Chrissy and Alessandra, Nico suggests changing the point of reference for sameness. He says "it [the parabola] bounces lower," meaning that they should think about sameness in relation to a different "lower" point rather than in relation to the origin. This shift in thinking is exemplified in other students' language; by the end of the video students abandon the language of sameness and begin using the language of "symmetry" coupled with a specific reference to non-zero vertex. They say, "it is symmetrical just not from zero." Students have refined their use of the intuitive notion of sameness to make it consistent with the mathematical concept of symmetry around any vertical line.