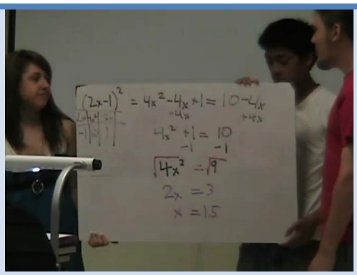
Solving a Quadratic Equation — Video Analysis Guide

Segment Focus	Approximate time in video	Line numbers in transcript	Visual Cues	Narrative Summary	Salient quotes
Initial solution	**0:00 – 0:50**	1 – 14		Students present their solution to the problem $(2x – 1)^2 = 10 – 4x$.	**Jeff:** “The square root of $4x^2$ is just 2x and the square root of 9 is 3.”
One or two solutions	**0:50 – 2:25**	15 – 52		Students discuss whether there should be two solutions to the equation.	**Ian:** “I remember something about you have to do plus or minus on one side.” **Jeff:** “2x could also be negative 3.”
Reverse order of operations	**2:25 – 3:58**	52 – 96		Students discuss whether the reverse order of operations was violated. Mr. Albritton asks the class to discuss the issues that have arisen in their groups.	**Luke:** “Doesn’t square rooting before dividing by 4 go against the reverse order of operations?”

Order of operations	**3:58 – 5:52**	97 – 129		Students discuss their ideas for whether the reverse order of operations were violated.	**Quinn:** “Technically we did violate them, but it doesn’t matter.”
Number of solutions	**5:52 – 6:54**	130 – 156		Students discuss whether the equation has one or two solutions.	**Nina:** “It’s symmetrical… If it’s 1.5 over here there’s going to be a -1.5 over here that corresponds to the same value.”

Solving a Quadratic Equation
Video Analysis Guide

Topic A: Jeff's Group's Solution

Central Issues:

What strategy did Jeff's group use to solve the equation?

They expanded the binomial on the left in order to cancel the x term. They isolate the x-squared term on one side of the equals sign and take the square root of both sides of the equation to solve for x. They include only the positive square root for the solution.

What was the group's thinking about whether or not they needed to expand $(2x+1)^2$?

They were hesitant to expand it at first because in some cases, the equation would be easier to solve without expanding the binomial.

What assumptions does the group make when they take the square root of both sides?

They consider only the positive square root and assume that is the only solution.

Relevant quotes include:

Lines 1-4
Jeff: For this one, the reason we didn't want to expand is, I mean, why would you want a squared there when you're looking for x? But if you expand 2 minus, $(2x – 1)^2$ in an area map like so, like the first, second group did, you get 4x squared minus 4x plus 1.

What does Jeff mean when he says "why would you want a squared there when you're looking for x?"

Additional Issues:

How do students respond to the group's solution?

Topic B: Taking the Square Root

Central Issues:

What ideas about taking square roots do students raise in the video?

See specific answers below.

Based on what Ian says, what do we think he understands about taking the square root of an equation?

Ian recalls the need to consider both the positive and negative values when taking the square root to solve an equation.

What idea does Damien raise about taking the square root of an equation?

Damien believes that taking the square root of both sides results in a plus or minus on both sides of the equation, thus canceling out the plus or minus signs.

Are Damien and Ian thinking about taking the square root in the same way?

They both talk about the plus and minus sign, but Damien thinks it's on both sides of the equation, whereas Ian seems to talk about it only being there once.

Both Ian and Damian see taking the square root as leading to a positive or negative value.

Ian thinks that taking the square root will create two answers, whereas Damien thinks that there will only be one.

<u>Relevant quotes include:</u>

Lines 33-34
Jeff: You, you're saying that 2x could also be negative 3 because if you square it it also comes out to be 9?

Lines 16-18
Ian: Uh, actually I kinda remember, uh, when we square root things and we're finding x's there was something about, like, plus or minus. And because there was x squared, it would be plus or minus that.

Lines 45-49
Damien: But wouldn't that, uh, wouldn't that work both ways? Well, like, if it's plus or minus for whatever is being square rooted and it's plus or minus on that side it would have to be plus or minus on that side, too. But then if both sides, plus or minus they kind of cancel out when we divide three by 2.

<u>Additional Issues:</u>

What's correct about Damien's thinking? Where does he run into a problem?

Damien is correct that there should be a positive or negative value but he believes they should be on both sides of the equals sign. Because Damien says they would cancel each other out, he seems to think that it would either be positive equals positive, or negative equals negative. He's forgetting about the combination where one side is positive and the other is negative. It's *interesting that if Damien hadn't made this error, he actually would have gotten the correct answer (x = ±1.5) even though he was thinking about the problem incorrectly (by having a plus or minus sign on both sides).*

Topic C: Order of Operations

<u>Central Issues:</u>

What is the main concern students have about order of operations?

The main concern is whether the reverse order of operations was violated.

Who believes order of operations is violated and what reasons do they give?

Nina says that 'to keep tradition' it's better to always follow the same order. Nina also says that we should follow the order of operations 'so we don't mess it up if we're in a different problem that it does matter in,' so she thinks that the reverse order of operations doesn't matter in this case, but that it sometimes will. In addition, Nina feels it's easier, or better, to have an algorithm to follow, rather than approaching each problem as if it's brand new.

Who believes order of operations is not violated and what reasons do they give?

Luke says that this is 'one of the exceptions' where the order they used was okay.

Who believes it doesn't matter and what reasons do they give?

Joe says that the order of operations doesn't matter. He recognizes that squaring is just a shortcut for repeated multiplication. Since squaring is really just multiplication, he feels that it doesn't matter which one you undo first. With this reasoning, Joe would probably believe that the order of operations never matters, not just in this problem.

Damien says that we can do whatever we want, as long as we do it to both sides of the equation. We don't have evidence about whether Damien sees any benefits to following the typical order of operations.

Jason implies that it doesn't matter because 'you'd still get the same thing.' Nina and Quinn also talk about the answer being the same with either order.

<u>Relevant quotes include:</u>

Lines 76-79

Damien: Well, would it really matter if you divided by... well dividing by 4 first wouldn't... doesn't really have a point, like isn't it just if you do something to one side, you do the exact same thing to the other side to make both sides equal.

Lines 101-103

Quinn: I think that technically we did violate them, but it doesn't matter because, uh, if you divided by 4 first then 9 divided by 4 is 2.25 and the square root of that is 1.5. It’s the same thing.

Lines 105-106

Luke: But this is only a specific example, where it's one of the exceptions. If we did a different problem, like would it work for all problems in general?

Lines 115-118

Nina: Well, if it doesn't matter, I feel like, just to keep tradition or just to make it so we don't mess it up if we’re in a different problem that it does matter in, you should stick to order of operations. I mean, the answer's still going to be the same, but it's probably a better method.

Lines 122-126

Joe: Um, well, if you square something, it's just multiplying it by itself twice so in, in the [inaudible] it's still a form of multiplication. So it doesn't really matter which one you do first, squaring or division because it's all under the same [inaudible]. So you just like prefer to get rid of the square first, because it makes things easier in some ways.

Additional Issues:

What about this particular problem might have led the students to use an atypical order of operations?

The perfect square coefficients may have led to taking the square root as the first step because they may have wanted to get the square root out of the way and avoid taking the square root of a fraction.

Topic D: One Solution or Two?

Central Issues:

At what points in the video do students discuss the number of solutions the equation has?

Student disagree about the number of solutions the equation has.

What justifications do students give for one solution?

Damien believes there is one solution because he thinks that taking the square root results in a positive or negative sign on both sides of the equation. He believes they will cancel each other out and there will only be the positive solution.

What justifications do students give for two solutions?

Ian recalls that taking the square results in both a positive and negative value on one side of the equal sign.

Nina is using the graph of the quadratic function to think about the number of solutions the equation has. She uses the fact that the quadratic graph has vertical symmetry and argues that it will intersect the x-axis twice, meaning that the equation has two solutions.

Becky confirmed for herself that the equation has two solutions by plugging both of the solutions back into the equation and seeing that they both work.

Relevant quotes include:

Lines 137-142
Nina: If you think about it like a graph. It's like, since this is sort of a quadratic equation, it might be, I'm just not really sure, but if it's a parabola (motions and traces out a parabola that opens downward) in a parabolic shape, then it's symmetrical. So you're gonna have the same point, like, not the same point, but if it's 1.5 over here there's going to be a negative 1.5 over here that corresponds to the same value.

Lines 132-133
Becky: Um, I substituted negative 1.5 and 1.5 back in the problem and they both worked out.

Additional Issues:

What different representations do students use in their justifications?

In what cases is Nina's reasoning correct?