Comparing Slopes
Explore Themes in Student Thinking

Throughout the video, students use a range of language and representations to talk about slope. In each video segment, one or more students describe their thinking about slope. Stepping back from the chronology of these ideas, we can see three main themes in the students' thinking. Students talk about their understanding of slope in terms of movement, the slope formula, and the special cases of horizontal and vertical lines.

Theme 1: Understanding Slope In Terms of Movement
During the discussion, some students focus on movement along a graph or line when describing their thinking about slope. In particular, students seem to equate slope of zero with a *lack* of movement. For example, Patrick explains that a horizontal line has a slope of zero because "it's never moving up." Arnas similarly focuses on movement when describing why the slope of a vertical line would be zero. He says, "if the origin is 0, 0 and it went like up and down and didn't like move at all, it would be 0."

It is interesting, and possibly surprising, that even though they are describing different lines (one horizontal and one vertical), Patrick and Arnas each use the idea of "no movement" to justify the "zero-ness" of slope. However, Patrick seems to refer to a lack of up and down movement and Arnas, while acknowledging movement up and down, refers to a lack of movement in another direction. Their use of the word "moving" may be an attempt to connect intuitive, everyday understandings with formal mathematical ideas they are learning in class about slope. However, the fact that "no movement" may be applied to either the x- or y-directions may make it difficult for students as they try to distinguish between the slopes of horizontal and vertical lines.

Theme 2: Understanding Slope In Terms of the Slope Formula
Rather than relying on an intuitive sense of how lines are "moving," at some points in the video students draw on their knowledge of the slope formula to reason about horizontal and vertical lines. For example, when Patrick explains why the slope of a vertical line is zero, he says, "since you subtract the change in x and the change in y, you subtract 0 and 0 and 5 and 0. So on the top there'd be 0 and on the bottom there'd be 5…it has to be zero." Patrick appears not to realize that he inverts the slope formula. Alfredo also uses a formula to calculate the slope but in contrast, believes that the slope is undefined because the zero would be in the denominator rather than in the numerator ("there can't be a number over zero").

As with the case of using "movement" to define slope, here again are two students who appear to use the same reasoning – the slope formula - to come up with different answers. What is not clear is whether Patrick's inversion of the formula is a simple calculation error (possibly supported by his previous reasoning about movement) or whether it is rooted in a larger confusion about slope as "run over rise" instead of "rise over run." As may often be the case, these student calculations reveal very little about their conceptual understanding of the concepts associated with those calculations.

Theme 3: Understanding Slope of Special Cases (Horizontal and Vertical Lines)
This class discussion focuses exclusively on slopes of horizontal and vertical lines. Their talk about those lines indicates that they may, at least tacitly group together those lines as "special cases" of slope. For example, early in the video Veronica explains that a line with zero slope is "a straight line." When asked by the teacher to sketch an air graph of the line, Veronica explains further "I just know it's a straight line. I'm not sure which way it's going [horizontal or vertical]." In this comment Veronica attempts to distinguish "straight lines" (i.e. horizontal and vertical) from "tilted" lines (i.e. lines of positive or negative slope), setting up "straight lines" as a special case for discussions of slope.

A tendency to group together horizontal and vertical lines as "special cases" may contribute to Patrick's claims, at different points on the video, that both lines have a slope of zero. It is unclear whether Patrick believes that both horizontal and vertical lines have slopes of zero, or if he is just reasoning about them independently. Either way, Patrick may see zero as a reasonable and attractive answer in both cases because of the fact that the two lines are both "special" and "different" from lines with positive or negative slope.