# Relating Perimeter and Area
## Explore the Mathematics

Consider the following problems:

1. If the perimeter of a square is doubled, what happens to the area of the square? If the perimeter of a square is tripled, what happens to the area of the square?
2. If the perimeter of an equilateral triangle is doubled, what happens to the area of the triangle? If the perimeter of an equilateral triangle is tripled, what happens to the area of the triangle?

In general, if the perimeter of a figure is multiplied by n, its area is multiplied by $n^2$. For example, if the perimeter of a square is tripled, its area is multiplied by 9. (Note that this relationship holds whenever the resulting figure is similar to the original figure, that is when the ratio of the sides does not change: when a square becomes a larger square, an equilateral triangles becomes a larger equilateral triangle, a rectangle becomes a similar rectangle, etc.)

There are a number of ways to justify this relationship. Here are several examples.

1. Calculating specific cases:

   A square- If a square has sides of length 3, its perimeter is 12 and its area is 9. If the perimeter is doubled, the perimeter is now 24, making the sides 6 and the area 36. The perimeter was doubled, and the area was multiplied by 4.

   An equilateral triangle- If an equilateral triangle has sides of length 4, its perimeter is 12. We can find its area by dividing the product of base and the height by two. The base is 4 and the height can be calculated by using either the Pythagorean Theorem or right triangle trigonometry (see image below). Doing so, we find that the height of the equilateral triangle is $2\sqrt{3}$, making the area of the triangle $4\sqrt{3}$. If the perimeter is doubled, the perimeter is now 24. The base is 8 and we can calculate the height as before to be $4\sqrt{3}$, making the total area $16\sqrt{3}$. This is 4 times the original area.

2. Using variables or a formula: Some teachers may choose to label a side of the figure "x" and then examine the relationship between the area and perimeter in general:

   For a square:

| Side | Perimeter | Area |
|---|---|---|
| x | 4x | $x^2$ |
| 2x | 8x (original times 2) | $(2x)^2 = 4x^2$ (original times 4, which is $2^2$) |
| 3x | 12x (original times 3) | $(3x)^2 = 9x^2$ (original times 9, which is $3^2$) |

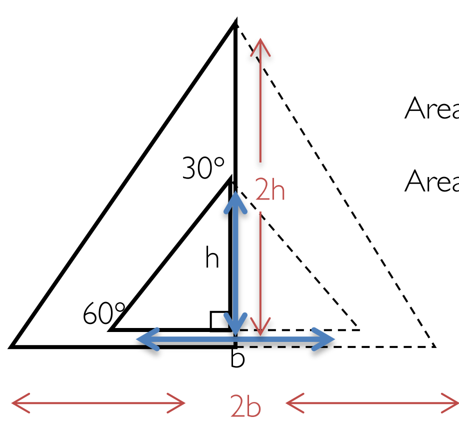
For an equilateral triangle:

| Base | Height | Perimeter | Area |
|---|---|---|---|
| x | $\frac{x}{2} * \sqrt{3}$ | 3x | $\frac{x^2}{4} * \sqrt{3}$ |
| 2x | $x * \sqrt{3}$ | 6x (original times 2) | $\frac{1}{2}(2x)(x * \sqrt{3}) = x^2 * \sqrt{3}$ (original times 4, which is $2^2$) |
| 3x | $\frac{3}{2}x * \sqrt{3}$ | 9x (original times 3) | $\frac{1}{2}(3x)\left(\frac{3}{2}x * \sqrt{3}\right) = \frac{9}{4}x^2 * \sqrt{3}$ (original times 9, which is $3^2$) |

3. Thinking about dimensions: Teachers may make the following argument: When you multiply the perimeter of a square by 3, you are essentially multiplying each of the dimensions by 3. To find the area, you would multiply the dimensions together. This means you'd end up multiplying the "times 3" by the "times 3;" in other words, you'd multiply the area by $3^2$.

   The dimensions of a triangle used to find the area are the base and the height. We can see that when all of the side lengths of a triangle are multiplied by a scalar factor, the height is multiplied by that same factor.

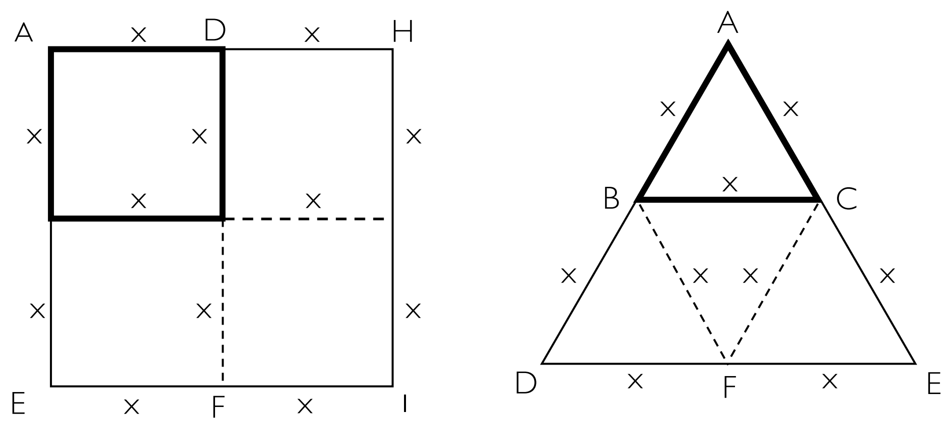
   We can think of an equilateral triangle as two right triangles. Suppose we double all of the side lengths of the equilateral triangle. Then the hypotenuse and base of each of the right triangles also gets doubled. Since we are multiplying two sides of each right triangle by the same factor, the resulting right triangles are similar to the originals. Thus, all of the side lengths are proportional, and since the base and hypotenuse were doubled, the height gets doubled as well. We can then use the same argument as we did for the square.




$Area_T = 1/2\ b*h$

$Area_{T'} = 1/2\ (2b)*(2h) = 4(1/2\ b*h) = 4(Area_{Original})$

4. Creating diagrams: Some teachers may draw diagrams that show how the increased perimeter affects the area. Notice that doubling the perimeter has created 4 squares (or triangles) the size of the original square (or triangle). In other words, doubling the perimeter quadrupled the area.




*Connecting to Algebra*
While on the surface this lesson appears to focus on geometry, it also involves algebraic thinking. Describing patterns and making generalizations are both fundamental to work with algebra; algebra is very much about shifting from specific examples to generally applicable methods, precisely what the students are doing in this video. In addition, the students are thinking about the functional relationship between perimeter and area. We can think of the increase in area as a function of the increase in perimeter.

*Connecting to the Common Core Standards*
A-CED2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF1 Write a function that describes a relationship between two quantities.

S.4. Model with mathematics.