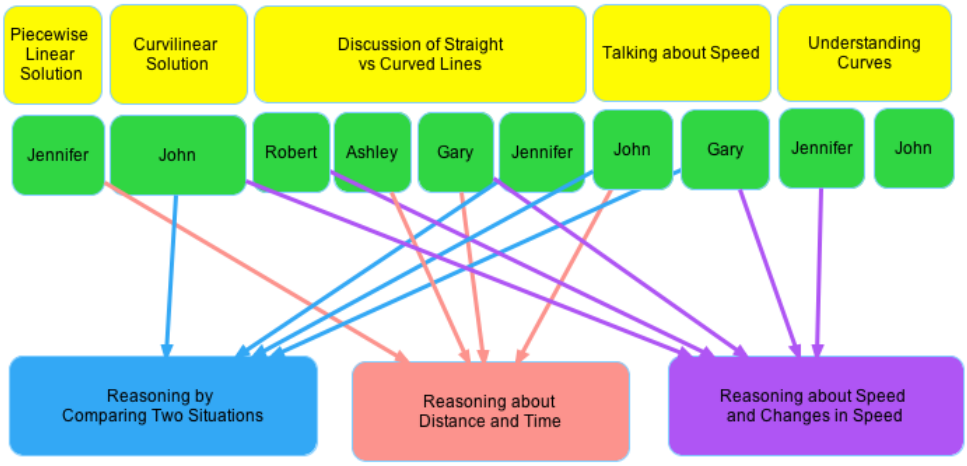
Graphing a Story: Straight vs. Curved
Explore Themes in Student Thinking

Throughout the video, students describe their reasoning for choosing either a linear or curved segment to represent the motion of the cargo ship between 2 and 4 hours. Stepping back from the chronology of these ideas, we can see three main themes in the students' thinking. Students reason across and within time segments of the problem, they reason about distance and time as two distinct quantities, and they reason about speed and changes in speed.



Theme 1: Reasoning by Comparing Two Situations
When talking about the graph for the cargo story, students in the video often use comparisons to reason about the shape of the graph. Some students make comparisons *across* the two time segments described in the story, from 0-2 hours and from 2-4 hours. For example, Jennifer's focus on the transition point, the two-hour mark, suggests that she is reasoning about differences between the two time segments. She talks about how the boat "accelerates from the, after the two hours, the first two hours." Gary similarly distinguishes between "the first segment… and the rest of it."

In contrast, other students make comparisons *within* a single time segment, usually from 2-4 hours. For instance, when describing the curved section John says: "the distance would go up more at the end and less at the front [of the 2-4 hours segment]." He later quantifies this difference; "it [the curved graph] starts out and goes about 25 [mph], [and] the other side [of the curved graph] goes 35." As evident in these examples, students often use pronouns such as "it" when referencing different parts of the graph for either comparison. As a result part of the difficulty in interpreting student thinking in this video involves identifying, sometimes from vague language, which two aspects of the story students are comparing.

Theme 2: Reasoning about Distance and Time
Students in the video often describe the different parts of their graph in terms of the distance and time the cargo ship has traveled, rather than focusing directly on the speed of the ship. This focus

is not surprising; the students are creating graphs of distance versus time so it is only appropriate they would discuss those two quantities. For example, John talks about how far the cargo ship travels in a given hour; "from hour 2 to hour 3 it goes up 3 ten's [on the graph]." Gary connects this change in distance over time to steepness by saying "you curve it to have more distance in less time." In these cases, focusing on distance and time seems to be useful for reasoning about the graph.

In other cases, focusing on distance and time as distinct quantities may support unproductive reasoning. For example, Jennifer's attention to the distance and times given in the problem - 20 miles in 2 hours, 80 miles in 4 hours – may lead her to a "connect the dots" type of graphing strategy. If she thinks all she needs to do connect the point (2, 20) to the point (4, 80), then the most direct way to do so is with a straight line segment. Ashley's focus on distance and time is problematic in a different way; she talks about distance in terms of the length of the actual segment on the graph, rather than the distance the cargo ship has traveled. She argues that the straight line segment "doesn't take as much time" as the curved segment because, in Mr. Louis' words, "there's less distance on the line."

Theme 3: Reasoning about Speed and Changes in Speed

Throughout the video, some students explain how the graph represents the speed of the cargo ship at different points. When describing the straight-line segment Robert says, "a straight line means it's going the exact same speed." Gary similarly states that a straight line "continues at the same speed all the way up." In contrast, John justifies his use of the curve by saying, "it goes faster and faster, so wouldn't it be curved?"

This focus on speed may make it easier for students to talk about changes in speed over time, or acceleration. For example, it allows students to discuss the central issue of whether the speed changes continuously or at a single point. Gary talks about continuous changes in speed on the curved graph by saying "it kept on accelerating all the way up… they were still accelerating." In contrast, Jennifer says "it speeds up then [at the two hour mark]," meaning that the graph "jumped up [once] and then didn't go like faster after that." This discussion about the change in speed (either continuous or instantaneous) is facilitated by the fact that both Jennifer and Gary understand the graph as representing speed as a single, distinct quantity.